

(All rights reserved)

ADVANCE PROOF—(Subject to revision).

This Proof is sent to you for discussion only, and on the express understanding that it is not to be used for any other purpose whatsoever.—(See Sec. 40 of the By-Laws.)

Canadian Society of Civil Engineers.

INCORPORATED 1887.

TRANSACTIONS.

N.B.—This Society, as a body, does not hold itself responsible for the facts and opinions stated in any of its publications.

(To be read Thursday 7th May.)

RAILWAY CURVES.

By HENRY K. WICKSTEED, M. CAN. SOC. C.E.

As railways have become more numerous, and have pushed their way into regions where traffic is thin or competition has become very keen, the study of the "economics" of operation has come to be more general among railway men and the influence of the different elements of location on the cost of running and maintenance better understood.

The earlier railways built through districts where trade was already established, and having only waggon roads to compete with, had merely to secure a good safe road at some reasonable cost, and their traffic and dividends were assured; but with competition, and in the case of roads running in advance of settlement, it became often necessary to secure the very greatest operative economy in the one case and the very lowest constructive cost in the other, in order to admit of the road's becoming a financial success.

There are those elements in the location of a railway between two given points, which can be utilized more or less interchangeably to reduce its cost below that of the straight line on plan and profile, which represents the perfect line—increase in distance, curvature and gradients, and all have a more or less detrimental effect on the operative value. In all roads they all three are made use of to some extent, but even at the present day there is evident a very marked difference of opinion as to their relative hurtfulness. In our older trunk roads, as in the English lines on which they were modelled, the first two seem to have been avoided at almost any cost. As nearly an air line as possible was obtained, and such gradients resorted to as would fit the ground without excessive work.

In the case of the English lines, the long rigid wheel base of cars and engines limited greatly the radius of the curvature, which could be used with safety and with very flat curves. Opportunities for reducing the pitch of gradients by lengthening the distance did not so often occur; but even since the universal adoption of the swivelling truck on all rolling stock on this continent, there still remained a marked repugnance to going even a very little way off the general heading to secure an easier line, and railways even of the very cheapest class were run up hill and down, almost as the crow flies; and to bring the cost within the means of the promoters, such gradients were adopted as have in many cases completely ruined their earning powers for all time to come.

Far be it from me to urge the promiscuous use of sharp curves and the practice of running assured every trifling obstacle, which is the opposite extreme towards which some engineers belonging to a later school have gravitated.

The best location for a road depends on many things: the purpose for which the road is built, the direction and volume of traffic, and the capital available, etc., etc., and no hard and fast rule can be laid down for general adoption. All I wish to point out is, that where, from financial weakness or other cause, it becomes necessary to choose between a somewhat circuitous route having easy grades and a comparatively straight one with heavy grades, the former will in a large majority of cases be the proper one to adopt, even if the difference in distance be quite considerable, and that where the choice is merely between sharp curves and heavy gradients, there can seldom be any reason for hesitation whatever.

The financial reasons for this are not difficult to understand. On a straight level track, train resistances at moderate speeds amount to about 6 lbs. per ton. On a 10° curve, this is about doubled, in other words, the increased resistance due to a 10° curve is only the same that we should have on a grade of $\frac{1}{10}$ per cent. or 16 feet per mile.

Yet how often we see grades of 60 to 80 feet per mile used in conjunction with curves of 3° or 4° , where the use of sharper curves would have made a longer rise at say one-half that rate practicable at the same cost and allowed of the train load being doubled. No railway manager needs to be told what an economy is effected by adding even 3 or 4 cars to his trains.

There are, however, other factors besides the low cost of hauling, which enhance the popularity and earning powers of a road: speed, safety and comfort in the running of trains and the cost of maintenance of track and rolling stock.

As regards the effect of curvature upon the first speed, 10° curves can be traversed and are traversed every day at 30 miles per hour; and with modern air brake equipment the loss of time caused by slackening the speed, from the ordinary maximum of 50 miles for an occasional curve, a few hundred feet in length, is very small, while even with ordinary passenger trains the effect of steep gradients in retarding speed is constantly felt. Freight trains very seldom travel more than the 30 miles per hour in any case, so that the curve does not affect them at all, whereas the gradients are all important to them as limiting the weight and length, and practically fixing the minimum remunerative tariff.

2nd. Safety.—No one will or can deny that curves are more or less an element of danger—both in increasing the liability to derailment and in obstructing the view of the track ahead; but both theory and practice tend to show that while a mathematically straight track is undoubtedly a great boon—a very desirable thing indeed—the tendency to danger on curves from both causes is (within moderate limits) almost independent of the radius of curvature. Centrifugal force increases inversely as the radius; but it is a fact not generally or, at any rate, not fully realized, that centrifugal force accounts only for a portion of the derailing tendency, the balance being caused by the necessity for the wheels to slip on the rail, both in a direction parallel to the axle—owing to the want of parallelism between the latter and the radius of the curve, and also in a longitudinal direction, owing to the outside wheel being obliged to roll further than the inside; this slipping or sliding friction is in accordance with the laws of friction generally and practically independent of the speed, and at very low speeds there is practically no more flange pressure against the side of the rail on sharp curves than on flat ones. Sharpen the curve to the extreme limit possible, and the wheel will at last mount the rail, not because the directive force exercised by the rail is greater, but because the obliquity of the plane of the wheel to the rail becomes greater than the angle of friction between the two.

Turnouts with curvatures of 6° and more are traversed daily and hourly by express trains, at speeds of 20 to 30 miles an hour on our best roads, although there are in the case of the turnout the additional elements of danger caused by the break in continuity of the outside rail at the frog, and the absence of superelevation to counteract the centrifugal force, as also that the turnout is nearly always very nearly a reversed curve, the tangent from the point of frog being seldom more than 30 or 40 feet. A well laid 8° curve on the open road is vastly safer at 30 miles than the turnout at 20. Yet we constantly see this anomaly of 6° turnouts traversed by the same trains on the same roads where on the open road a 2° or 3° standard has been enforced at a heavy cost.

With regard to the obstruction of view by a curve, it needs no demonstration to shew that a rock bluff or building close to the track will limit the range of vision on a 2° curve to within a few feet of what it is on a 10° ; and unless we can eliminate the curvature altogether, we can gain little in this respect by lengthening the radius. There are without doubt many accidents attributable more or less directly to curvature, but in very few of those which I have had an opportunity of inquiring into does it seem to me that the radius of curvature had much effect, and in many it would appear that some defect in the roll-

ing stock was the *direct* cause, although perhaps even with this defect existent the accident would hardly have occurred elsewhere than on a curve. The recent accident on the Intercolonial at St. Joseph, for instance, is a case in point, and appears to me distinctly traceable to the manner in which the brake beams and shoes are generally hung on passenger cars. One shoe presses the front side of the leading wheel, and in a majority of cases at a point a little below (sometimes far below) the axle, the friction of the wheel tends to lift it; and once lifted, if the brake is below the centre, it cannot drop again until the brake is released. The brake beams are again hung from the body of the car, not from the truck. Now, imagine the car body inclined, owing to super-elevation of the track, and bearing rather more heavily on the inside springs and compressing them, the brakes are applied and the wheel is clamped in this position. Before they are released the cant is reversed and the wheel is lifted, or at any rate the pressure on the rail is so reduced that it mounts the rail with great ease. The author has so often seen this lifting action where he has watched wheels to which the brakes were suddenly applied with full pressure, that he has no doubt whatever that this was the cause of the derailment in question, as of many other similar ones. We generally read in the evidence that the brakes have been applied, and that within a few seconds the derailment occurs. In the case of engine derailments we seldom or never hear of the leading truck getting off,—there are no brakes on it,—seldom or never of the leading truck of the tender leaving the rails, the close coupling to the engine prevents this; but the rear truck is constantly being derailed, and very often from the cause pointed out, and which, by the way, was very ably discussed, and the remedy pointed out in a recent issue of the *Engineering News*. So much for the elements of danger *inherent* in curvature.

On the other hand, some of the most terrible accidents which have ever appalled us have been directly caused by gradients. The Irish disaster at Armagh is one of the most destructive, and a similar one occurred some years ago on the Southern Pacific. These are directly chargeable to heavy grades, and a number of others are indirectly chargeable to the same cause in this way. A train is timed to make, say, 45 miles per hour between stations on a road having heavy undulating gradients on the up grade—this is, let us assume impossible—and the speed falls to 25 miles at the head of it. Assuming the next summit to be of the same elevation, the speed at the bottom of the sag must be in order to keep up the average 65 miles per hour, this is so easy to attain on the down grade that it is very common indeed; but when, as is often the case, the line is complicated at the bottom by curves, structures, or by frogs and switches, it is by no means a safe speed, and the consequences are often disastrous. Breaking away is also a fruitful cause of trouble on steep grades.

3rdly. Comfort.—Probably the most serious argument of all against the use of sharp curves as commonly introduced is the discomfort which they cause to passengers (especially to those in sleeping cars) when traversed at high rates of speed. Travelling over a long stretch of one of our trunk roads not long ago, the author was several times hurled across the car while endeavouring to dress and the impact was so violent at times that it was difficult to persuade himself that there was not something of danger as well as discomfort.

Steep grades, on the contrary, whatever may be their effects on the pockets of the shareholders or the safety of the passengers, do not influence the comfort of the latter, and the average one merely wonders why the train is moving so slowly at the top of a grade, or experiences a slight exhilaration when it nears the bottom, and that is all about it.

Lastly, maintenance of way and wear and tear.

There is certainly quite a difference in this respect between the curve and the straight line, but there is also a great difference between the grade and the level and between the heavy grade and the light creeping and buckling of rails, use of sand to aid adhesive broken links, frequent application of brakes, all tend to keep both the roadmasters and the mechanical departments busy, and run up the cost of repairs. And here again we find that it is *curvature* "per se" which makes the trouble, and the *radius* of curvature has a comparatively small effect. In other words, having 90° of curvature, it makes little difference whether it is present as 3000 feet of a 3° curve or as 900 ft. of

a 10° , and it is constantly maintained that as far as repairs to roads are concerned, 3000 ft. of 3° curve is a little more expensive than 900 ft. of a 10° , and 2100 ft. of tangent. Without asserting this, the author is content to believe that under ordinary circumstances they are nearly equal.

We have seen then above that only in the item of comfort to passengers is there much to be said against the use of much sharper curves than those in ordinary use, say up to 8° or 10° .

Whether the author is right or wrong in his conclusions as above, it is certain that the tendency is towards their use and towards flattening the grades as much as possible. This is a money-making age, and whereas the question of grades is one of dollars and cents from first to last. The objection to curves is admitted even by those most opposed to them to be largely a matter of sentiment. The average business man will buy an accident ticket, and risk his life on the crookedest of roads, but he will not invest his money in a road which, owing to the cost of operation, will not pay dividends.

When money is scarce, and heavy work cannot be resorted to, to reduce gradients, we have no choice, therefore, very often between a crooked road and no road at all. And there is this strong argument on a new road in favour of adopting sharp curves rather than heavy grades, that having a rise of say 320 ft. in 4 miles or 80 ft. per mile, connecting two level or nearly level stretches, it is impossible to reduce this hereafter to 8 miles of a 40 ft. grade—otherwise than by the complete abandonment of at least 8 miles of road,—no part of the original road will serve for the altered one. Whereas a few sharp curves can often be taken out or reduced by gradually cutting into a hillside at one point, and filling out at another; and the cost of doing this, distributed over several years, during which the road is earning money, will scarcely be felt, although it would have been serious and perhaps prohibitive if undertaken in the first place.

This practice of opening roads at the earliest possible moment, and improving them as growing traffic warrants the expense, has become so very common and is so eminently reasonable and sensible, that this is in itself in many cases sufficient reason to decide the choice between the two hurtful elements.

In a majority of lines the traffic is light in the first place, and gradually increases. Hence, for the first few years we must endeavour to keep the capital cost and fixed charges as low as possible, they being the heaviest charges against the revenue. As the traffic grows the maintenance account grows almost in the same ratio, and becomes of greater relative importance, hence we are justified in increasing the capital account to diminish it.

Without further discussion as to why and wherefore, when and where they should be used, which is a little foreign to the purpose of this paper, the author proposes to investigate supposing sharp curves to be adopted, what their form should be, and how we may best make them as little hurtful and as comfortable to ride over as may be.

To overcome the evil effects of centrifugal force it is the common practice, as every one knows, to elevate the outer rail to an amount which varies with the speed intended. This amount is generally put down at .05 feet for every degree of curvature corresponding to a speed of 30 miles per hour, and the results are good for the ordinary range of curves. The author's own experience is that this gives rather too much for the very sharp curves, which are run over at low speeds, and not enough for the flat ones; but for curves from 1° to 8° , this rule is as good probably as any.

Once the car is on the curve, this elevation or cant is quite effective in counteracting the centrifugal force, and whether slightly too much or too little matters little, as the body quickly adjusts itself a little out of the perpendicular to correct the deficiency or surplus. With 6 inches elevation we may ride around a 10° curve at 30 miles an hour, and experience neither danger nor discomfort; but if we change abruptly from a straight line to a 10° curve, we must either have the track insufficiently elevated on the curve, or have 6 inches of elevation on the tangent, and in either case we have a sharp jerk as the car strikes the curve, after which it travels steadily enough until it leaves it again, when another minor lurch is experienced.

The reason for the harder lurch at the beginning of the curve, the

author has never seen explained, except as the effect of insufficient elevation at the point of curvature; but an additional cause is as follows:

On the tangent the truck frame is parallel to the axis of the car, and both wheel flanges are travelling clear of the rail, and generally owing to play, slack gauge, and worn flanges an inch or more from it. A very simple calculation will show that with this 1 inch of play the truck will run straight on at 10° curve for some 10 feet before the flange will come in contact with the rail, and the change in direction is then angular instead of circular, hence the lurch which no amount of superelevation will counteract. It is on this account that widened gauge and derailments almost invariably occur at the ends of curves. There is not only an angular change in the motion of the car far more marked, than at any other portion of the curve, but there is the impact of the flange against the rail, and the sudden almost instantaneous change in direction of the truck itself with reference to the car body. The want of freeness in swivelling motion of the truck, owing to an excessive heading on the friction blocks or other cause, is another very fruitful cause of derailment.

To obviate this shock at the ends of curves, various forms have been proposed—parabolic, logarithmic and others; but these are not only so cumbersome to lay out as to be almost useless in practical location, but do not always effect the desired object.

It will be seen from the foregoing that the perfect curve is one in which the radius gradually and regularly diminishes from infinity to the minimum in such a distance, that the superelevation is not accomplished by a steep rise or fall in either rail. This minimum radius and maximum elevation is then retained to within the same distance of the end, when the reverse process takes place. At a gradient of 1 in 600 the maximum for a 10° curve of 0.5 feet will be attained at 300 feet, and at every 30 feet .05 feet corresponding to 1° curvature should be gained. The degree of curvature at any point on this 300 feet should be directly proportional to its distance from the point of curvature. Call the latter P. Fig. 1, and the ends of the several 30 ft. Chords. P_1, P_2, P_3, P_{10} , then the curvature is represented, at any of them by its corresponding index number. $1^\circ, 2^\circ, 3^\circ$ etc. This curve is luckily not only theoretically the best for the purpose, but easy to run in on the ground, either with the transit or by offsets from the tangent and from the final curve produced, and the effect on the easy riding of the car is marvellous. Any length of chord may be used, the longer for flatter curves, on which high speed is contemplated, the shorter for sharper curves than 10° , when it is difficult perhaps to get sufficient room for the longer. For ordinary practice the 30 feet or 1 rail length will be found very convenient for use.

Let P, P_{10} in Fig. 1 be such a curve uniformly accelerating from 0° at P to 10° at P_{10} whence it remains constant. The mean curvature will be 5° , and the length being 300 feet the total deflection is 15° represented by the angle T. 1 S. In like manner it may be demonstrated that the total deflection at P_2 is $1^\circ \times 6 = 36'$ at P_3 $144'$ or $2^\circ 24'$ in fact proportional to the squares of the index numbers.

It can also be demonstrated that the tangential angles $T_1 P, P_1 T, P_2 P_1$, etc., $T_1 P, P_1 T$ are $\frac{1}{2}$ of the total deflections, and are also proportional to the squares of the index numbers, the series being $3', 12', 27', 48'$, etc., up to 5° .

Here then we have the means of running in the curve with equal ease to a circular arc, setting the transit at P, and turning of the angles corresponding not to the distances themselves but to the squares of those distances arrived at P_{10} we put in a hub, move the transit up, and sighting back at P turn off 10° to the tangent $P_{10} S$, whence we can run the 10° curve in the ordinary way.

Arrived at the other end of the curve, the process is quite simple, although a little more figuring is involved.

Suppose we have run the curve round to P_{10} from C, and we wish to run the transition curve from P_{10} to P, producing the 10° curve towards P, it will be seen at once that the transition curve leaves it at precisely the same rate as it leaves the tangent in starting from P, and the series of angles will be the difference between those for a 10° curve and those given.

P_1 will be $5^\circ \times .3 = 1^\circ 30'$ less $3' = 1^\circ 27'$ (I being $^\circ$)
 P_2 " " $5^\circ \times .6 = 3^\circ$ " $12 = 2.4^\circ$
 P_3 " " $5^\circ \times .9 = 4^\circ 30'$ " $27 = 4.03$

etc., etc. etc., etc.
 P_5 " " $5^\circ \times 3 = 15^\circ$ " $5^\circ = 10^\circ 00'$

If it be the tangent to the 10° curve produced, until parallel to tt , it will be evident that the point of tangent is opposite P_1 , or midway of the transition curve, and from the foregoing that P_3 is equidistant from the circular curve and from the tangent on either side of it. Mathematically, this is not exactly true, but for the purpose we are considering it is practically so. A little consideration will show that as in the circular curve the offsets from the tangent are proportional to the squares of the distances from the origin, so in this curve we are considering the corresponding offsets from TT to the point P_1, P_2 etc., are proportional to the cubes of the distances or to the cubes of the index members, and correspond to the series 1, 8, 27, 64, etc., 1000, the tangential offset for P_1 can be calculated to be .02615, hence the far P_2 it will be $.02615 \times 125$ or 3.27, and the rectangular distance will be between TT and tt , double of this, or 6.54.

This property of the curve under discussion has caused Mr. Wellington, who first called attention to it, in print to designate it the "cubic parabola."

The circle being the "simple" curve, having the tangential angles proportional to the distances, this curve might, from the peculiarity mentioned above, be called the "quadratic" curve.

As the parabola is practically coincident with the circle for small arcs, so the cubic parabola is practically identical with the quadratic curve for small arcs, and the formulæ for either may be used interchangeably on transition curves of small total angle; but if carried on, the two will soon separate widely, as will the circle and parabola with the same initial radius of curvature.

This property of offsets suggests another method of running in the curve, which in certain situations is easier and more expeditious than the preceding.

Run the 10° curve round to the end at P_3 , offset the tangent 6.54 ft., put in P on this tangent 50 ft. from P_3 , and then offset the stakes P_1 to P_4 from the tangent and P_6 to P_5 by corresponding amounts from the curve.

In some situations, and notably where it is required to readjust old curves without incurring too much work, the displacement of $6\frac{1}{2}$ feet will be found too great. In such cases it should be remembered that with chords of even half the length assumed, or 15 feet, there would be a rise in the outer rail of only 1 in 300, which is by no means excessive. Dividing the chord by 2 has the effect of reducing the total displacement to $2\frac{1}{2}$, $\frac{1}{4}$ of the original, or 1.6 feet, an amount which can generally be easily attained, half the displacement being placed on the curve and half on the tangent. To avoid cutting rails and altering the length of old track, the central portion of the curve should be sharpened and the crown thrown slightly outside that of the original curve. Fig. 2.

Let us suppose, Fig. 3, that we have a very common location problem—two fixed tangents intersecting at a given angle—it is required to fix the point of curvature P on each, of a curve of given radius connected with the tangents by transition curves.

Let P, T and P_1, T_1 be the given tangents intersecting at the point I with an angle of say 60° , it is required to connect them with a 10° curve tapered at the ends by transition curves of 300 ft. in length, as in Fig. 1.

We wish to find the distance P_1 or the subtangent of the curve. We have seen that the total displacement of the tangents from those of the simple curve produced is 6.54 ft.; produce the central curve both ways, and draw the parallel tangents p, t and p_1, t_1 intersecting at i , draw the middle radius I, i, c_1, c , and let fall a perpendicular ia from i upon P, T and p, b from p upon P, T , then

$$PI = Pb + pi + Ia \quad Pb \text{ is by Fig. 1} = 150 \text{ feet.}$$

$$Ia = ia \times \cotan \frac{1}{2} 1 = 6.54 \cotan 60^\circ = 3.8 "$$

$$\text{and } -pi = 573 \times \tan \frac{60^\circ}{4} = 573 \times \tan 30^\circ = 330.8 "$$

therefore P, I

$$= 484.6 "$$

It will often be desirable to know how far the tapered curve fall inside the simple curve, connecting the same tangents represented by C, C in the figure; in other words, what is the difference in the crown distance $Ic, = ic$, since they both represent the crown distances for two

curves of the same radius, and having the same central angle; therefore

$$CC_1 = li = ia \sin \frac{1}{2} P.I.P. = 6.54 \operatorname{cosec} 60^\circ = 7.55 \text{ feet.}$$

A number of other problems might be given similar to those in the field books for circular curves; the figuring in every case is rather more complex than for simple curves, but the instrumental work is no more difficult, and once the reader has mastered the general characteristics of this form of curve he will readily solve them for himself, and probably be able to demonstrate the correctness of the results much more clearly than the writer.

The author wishes to say further that with compound curves the *módus operandi* is entirely similar, the tangential angles for each chord point being turned off from the corresponding points on the simple curve produced instead of from the tangent.

Suppose we are running a transition curve between a 6° and a 10° curve, the difference is 4° and the angles are precisely the same as those between a tangent and a 4° curve.

Let $P_1 P_2 P_3 P_4$ be the chord points, 30 feet apart as before. Then the angles from the tangent at P_1 the end of the 6° curve, will be those for the curve produced + those for the transition curve.

$$\begin{aligned} P_1 - 3^\circ \times .30 + 0^\circ.03' &= .54' + 3' = .57' \\ P_2 - 3^\circ \times .60 + 0^\circ.12' &= 1^\circ.48' + 12' = 2^\circ.00' \\ P_3 - 3^\circ \times .90 + 0^\circ.27' &= 2^\circ.42' + 27' = 3^\circ.09' \\ P_4 - 3^\circ \times 1.20 + 0^\circ.48' &= 3^\circ.36' + 48' = 4^\circ.24' \end{aligned}$$

Moving the transit up to P_1 the angle between PP_1 produced and the tangent at P_1 will be

$$3^\circ \times 1.2 + 0^\circ.48' \times 2 = 3^\circ.36' + 1^\circ.36' = 5^\circ.12'$$

In running a long curve over rough ground, it will often happen (perhaps owing to unavoidable errors in taking the notes off the platted plan) that the final tangent does not lie quite in the position we wish, but parallel to it. With a simple curve we should either start back to the beginning and try it again, or sharpen or flatten the end a little to bring it into position. The first course consumes much time, and the last state of that curve is sometimes nearly as bad as the first. The second method savours of patch work, and is never quite satisfactory. The transition curve, on the contrary, affords a means of adjusting such little differences with great accuracy, leaving the curve theoretically as perfect as if it had come out right in the first place.

Suppose again we have run the curve in Fig. 1 out to the end at P , and we wish to shift the tangent $2\frac{1}{2}$ feet further out or away from it. The offset it was 6.54, we wish to make it 9 feet. We have merely to increase the chord length in the proportion of the square roots of these offsets, C and O being the original, and C_1 and O_1 the new chord and offset;

$$\text{then } \frac{C_1}{C} = \sqrt{\frac{O_1}{O}} = \text{and } C_1 = C \sqrt{\frac{O_1}{O}} = 30 \sqrt{\frac{9}{6.5}} = 35$$

nearly, the initial tangential angle will be 3.5° the total length of transition curve 350 feet, and the total deflection $3.5 \times 3 \times 10^\circ = 1050'$ or $17^\circ 30'$ instead of 15° we have merely then to set P_{10} in 25 feet further back on the curve, and proceed with the new curve in the same way as before.

It will be seen that tapered curves are considerably longer than simple ones with the same central angle, and consequently in readjusting a location having sharp curves of contrary flexure separated by short tangents, we shall have to still further reduce these tangents. This, however, matters not, for the objection to absolute reversed curves disappears altogether when they take this form, calling angles to right + and to left — in the reversed curve the curvatures at the successive chord points are $4^\circ 3^\circ 2^\circ 1^\circ 0^\circ -1^\circ -2^\circ -3^\circ$, etc., differing always by unity as before. In Fig. 4 we have a reversed transition curve between two 5° curves of contrary flexure with the same pitch as in Fig. 1. Merely remembering that we are starting from a 5° curve instead of a tangent, we can set up on P and run through to P_{10} just as easily as we could the constant curve in Fig. 1. The change in curvature is the difference between 5° and -5° or 10° , consequently

the total length and total offset are the same in both cases, the tangential angles at P will be

$$\left. \begin{aligned} P^1 \quad 2^\circ 30' \times .3' - 3' &= 45 - 3' = 42' \\ P^2 \quad 2^\circ 30' \times .6 - 12' &= 1^\circ 30' - 12' = 1^\circ 18' \\ P^3 \quad 2^\circ 30' \times 1.5 - 1^\circ 15' &= 3^\circ 45' - 1^\circ 15' = 2^\circ 30' \\ P^4 \quad 2^\circ 30' \times 3 - 5^\circ 00' &= 7^\circ 30' - 5^\circ 00' = 2^\circ 30' \end{aligned} \right\} \begin{array}{l} \text{All positive} \\ \text{or to Right.} \end{array}$$

And moving up to P⁴ we turn off for the tangent of the 5.° curve at that point $2^\circ 30' \times 3 = 7^\circ 30' - 10' = 2^\circ 30'$ or $2^\circ 30'$ to left.

The author fears he has already exceeded the limits which such a paper as this should occupy, and will content himself with some general remarks on the treatment of curvature, generally without any further mathematical investigation which he feels that a large number of the members of the Society are better qualified to make than he.

Where grades and curves occur in conjunction, it is good practice in location to flatten the former for the distance over which such curves extend by an amount sufficient to neutralize the extra resistance. On a 10° curve therefore the grade should be flattened $\frac{1}{10}$ or thereabouts per 100 ft. In all discussions of this question the train moving upwards only has been considered. Mr. A. M. Wellington, for instance, who has written more ably and exhaustively on location questions than any one who has preceded him, quite neglects to take into account effect of this curve compensation on the swiftly moving downward train. He shows as others have before, and as any practical railway man knows for himself, the bad effects of a sudden change of gradient from steep to flat, in causing a "piling up" of the rear cars of a long train against the forward ones, which are meeting with extra resistance, often causing derailment even on straight track; and he strongly insists on long vertical curves connecting these grades of varying pitch.

Now, with compensated curves, this retardation is doubled, for the engine entering the curve on its downward path experiences not only the loss of the accelerating force due to the $\frac{1}{10}$ per cent. grade, but the resistance of the curve as well, equivalent to another $\frac{1}{10}$ or $\frac{1}{10}$ in all, and the tendency to derailment is much more than doubled on account of the curved position of the train tending to force the center cars towards the outside of the curve, or in the same distance in which there is already a strong disposition owing to centrifugal force and the want of radiality in the exles. Here, the author thinks, is another fruitful cause of wreck, to freight trains especially, which has not been fully investigated before, and which has led to an unnecessary prejudice against curvature *per se*—unnecessary, because the cure is easy. And here again is another argument in favour of transition curves. Able authorities contend that the rate of change in the gradient should not exceed $\frac{1}{10}$ per 100. The total change being, as we have shewn, equivalent to $\frac{1}{10}$ per cent., the vertical curve would have a length of 300 ft., or precisely the same as the transition curve we have been considering; and not only is the accelerating force urging the downward train gradually diminished, so that its loss is un hurtful, but the resistance to the upbound train is at all points precisely the same, and we have neither in plan nor profile any sharp angles or transitions from tangent to curve or from steep grade to flat.

As to superelevation of outside rail, it is generally conceded that it is better to carry this to the full amount which theory demands for the highest speed practicable on the curve, and that it is better to have too much elevation than too little. The author's own opinion is that the ordinary rule of $\frac{1}{8}$ ft. for each degree corresponding to a 30 mile speed is rather too little for the flat curves from 0 to 4°, about right at 6°, and altogether too much for sharp curves from 10° to 20°, and that an excessive elevation is more dangerous than the reverse, causing as it does at low speeds a diminution of weight on the outside wheel, which is the only one which has anything to do with directing the train, and consequently rendering it easier for the flange to rise over the rail.

It should be remembered, too, that while the tangential tendency of the outside wheel merely causes grinding action of the flange against the side of the rail, and strains internal to that rail itself, the inside wheel is dependent on the thrust of the axle for guidance, and through

both cases, the tan-

18' } All positive
30' } or to Right.
30' }

ent of the 5.° curve
2°30' to left.

limits which such a
itself with some gen-
erally without any
feels that a large
qualified to make

it is good practice in
r which such curves
extra resistance. On
ed to, or thereabouts
the train moving up-
ington, for instance,
n location questions
s to take into account
ly moving downward
nd as any practical
f a sudden change of
p" of the rear cars of
e meeting with the
right track; and he
ting these grades of

on is doubled, for the
periences not only the
grade, but the resis-
to or f_0 in all, and the
ed on account of the
center cars towards
ce in which there is
force and the want
ss, is another fruitful
ch has not been fully
nnecessary prejudice
he cure is easy. And
sition curves. Able
e gradient should not
we have shewn, equi-
have a length of 300
e we have been con-
urging the downward
s unharful, but the
s precisely the same,
o angles or transitions

generally conceded that
which theory demands
and that it is better
author's own opinion is
e corresponding to a
curves from 0 to 4°,
sharp curves from 10°
e dangerous than the
minution of weight on
has anything to do
dering it easier for the

tangential tendency of
of the flange against
rail itself, the inside
guidance, and through

it is constantly exerting a force tending to pull the
from this point of view also, it is better to have at least an equal
weight on both. The author has found also on sharp curves, only
intended and used for low speeds with a high elevation, that there is a
tendency in the track to move inward, the ties sliding on the ballast.

Widening the gauge contributes to ease in traversing curves, and
in the sharper ones is absolutely necessary; on a 10° curve $\frac{1}{4}$ of an
inch is sufficient, but in order to obviate the shock which the author
has dwelt on above, in entering the curve, the gauge should be light
and close at either end; with transition curves, it is easy to widen the
gauge uniformly from the beginning of the curve to the point of
constant curvature. Lastly, it cannot be too strongly insisted on that
curves should be run accurately with the instrument, and centers placed
permanently at short intervals, with fine brass tacks in their heads,
marking the exact position of the line. The track gauge should have
a notch marking its center, so that the foreman can at a glance see
whether the track is centered or not, without the use of the tape, and
the track level should have a sensitive bubble and an attachment for
getting the superelevation exactly.

The outside rail should be kept equidistant from the center, and any
variation in width of gauge thrown altogether on the inside rail. The
folly, nay, wickedness, of leaving the lining of curves to the foreman's
eye, and the elevation to his inner consciousness, is just as gross and
unpardonable as would be regulation of the depth of cuttings and
height of bridges by the same standard, and is very much more likely
to lead to accident. Centers should never be more than 100 feet apart,
and on curves never more than 50 feet; on 10° curves they should be
placed at least every 25 or 30 feet, and more frequently on the sharper
curves.

On a 20° curve, which the author ran in within the last few months,
and which has been in constant use ever since by passenger trains, the
centers were placed every 10 feet, and rigidly adhered to; the gauge
was widened 1 inch, and the elevation is $\frac{1}{2}$ inches, which is rather too
much than too little. No spreading of the gauge has taken place, and
on the greater portion not one dollar has been spent in repairs; such
repairs as have been done were on account of subsidences of new bank,
and they would have been equally necessary on tangent. Not the
slightest difficulty or expense has been experienced in operating this
curve for a year, either summer or winter. A stop is necessary for all
trains within a few feet of one end, and all trains therefore traverse it
very slowly; but the author would not hesitate to run over it at 15 miles
per hour. While not advocating the use of such extreme curves on
the open road, except temporarily and in extreme cases, the author
feels satisfied that properly laid, adjusted, compensated, and tapered
curves of 10° or even more might have been used to save money with
much better results as to safety and economy, in many cases where heavy
grades have been adopted to the same end, and that a great deal of the
prejudice which is shown, especially among the older men of our pro-
fession, against the free use of it, to save the capital account and increase
the operative value, is to a large extent sentimental and groundless;
and the author will be glad if this paper suggests anything worthy
of discussion or comment, whether favourable to his views or the
reverse.

STATION.	TANGENT L. ANGLE.	TOTAL DEFLECTION.
P ¹	0.45'—0.03' = 0.42'	1.30—0.09' = 1.21'
P ²	1.30'—0.12' = 1.18'	3.00—0.36' = 2.64'
P ³	2.15'—0.27' = 1.88'	4.30—1.21' = 3.09'
P ⁴	3.00'—0.48' = 2.52'	6.00—2.24' = 3.76'
P ⁵	3.45'—1.15' = 2.30'	7.30—3.45' = 3.85'
P ⁶	4.30'—1.48' = 2.82'	9.00—5.24' = 3.76'
P ⁷	5.15'—2.27' = 2.88'	10.30—7.21' = 3.09'
P ⁸	6.00'—3.12' = 2.88'	12.30—9.36' = 2.94'
P ⁹	6.45'—4.03' = 2.42'	13.30—12.09' = 1.21'
P ¹⁰	7.30'—5.00' = 2.30'	15.00—15.00' = 0.00'

STATION.	Dist. in ft.	Curvature	TANG. ANGLE	TOT. DEFLECT'N.	TANG. OFFSET
P ¹	0	0	0'	0'	0'
P ¹	30	1°	3'	9'	0.03'
P ²	60	2°	12'	36'	0.21'
P ³	90	3°	27'	1°21'	0.71'
P ⁴	120	4°	48'	2°24'	1.67'
P ⁵	150	5°	1°15'	3°45'	3.27'
P ⁶	180	6°	1°48'	5°24'	5.65'
P ⁷	210	7°	2°27'	7°21'	8.97'
P ⁸	240	8°	3°12'	9°36'	13.39'
P ⁹	270	9°	4°03'	12°09'	19.06'
P ¹⁰	300	10°	5°00'	15°00'	26.15'